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① (i) $(a^b)^c = a^{bc}$ since you've multiplied together b copies of a a total of c times, for a total of bc copies of a.

(ii) $a^b \cdot a^c$ since if we multiply together b copies of a, and then c more copies of a, that's a total of $b+c$ copies.

(iii) Rephrases (i) in terms of logs.

(iv) Rephrases (ii) in terms of logs.

② All false!

③ $f(x) = \sin^2(x) + 1$ $\quad f'(x) = 2\sin(x)\cos(x)$

$f\left(\frac{2\pi}{3}\right) = \sin^2\left(\frac{2\pi}{3}\right) + 1$ $\quad f'(x) = 2\sin\left(\frac{2\pi}{3}\right)\cos\left(\frac{2\pi}{3}\right)$

$= 3/4$ $\quad = 2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{-1}{2}\right) = \frac{-\sqrt{3}}{2}$

$$f(x) \approx \frac{3}{4} - \frac{\sqrt{3}}{2}\left(x - \frac{2\pi}{3}\right)$$

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⑤ (iii) $f(x) = x^{12}$ $\quad f'(x) = 12x^{11}$

$f(1) = 1$ $\quad f'(1) = 12$

$f(x) \approx 1 + 12(x-1)$

$f(1.01) \approx 1 + 12(0.01) = \boxed{1.12}$

(iv) $f(x) = \cos(x^\circ) = \cos\left(\frac{\pi x}{180}\right)$ $\quad f'(x) = \frac{-\pi}{180}\sin\left(\frac{\pi x}{180}\right) = \frac{-\pi}{180}\sin(x^\circ)$

$f(45) = \cos(45^\circ) = \frac{\sqrt{2}}{2}$ $\quad f'(45) = \frac{-\pi}{180}\sin(45^\circ) = \frac{-\pi\sqrt{2}}{360}$

$\cos(x^\circ) \approx \frac{\sqrt{2}}{2} - \frac{\pi\sqrt{2}}{360}(x-45)$

$\cos(46^\circ) \approx \frac{\sqrt{2}}{2} - \frac{\pi\sqrt{2}}{360} = \boxed{\frac{(180-\pi)\sqrt{2}}{360}}$

This is correct to four decimal places!